Progression in Reasoning

This progression map is written to help teachers to meet the aims of the national curriculum for maths, and in particular to promote mathematical reasoning in children, to develop an ability to convince others using mathematical arguments and to engage with non-routine problems.

Routine problems, as in closed worded problems, are included in the National Curriculum programmes of study and therefore are not included in this progression map.

The progression map presents four different types of mathematical reasoning, although these are linked and often overlap and teachers can still see the progression in each type of reasoning.

Mathematical reasoning and problem solving are best embedded in all lessons and the learning outcomes and activities suggested here are recommended as part of everyday teaching, rather than as discrete problem-solving lessons.

When teaching children to convince others and engage with ideas of proof, teachers can:

- Use ideas across the mathematics curriculum, to ask children to convince others of facts and ideas. In particular, they may use resources such as Dienes and place-value counters to show how they have reasoned about number. Some examples are listed in this progression map.
- Ask children to discuss general statements and argue whether these are true or not true. To show something is true they might first look at some examples to convince themselves or others but as they move through Key Stage 2, they can present an argument based on the properties of numbers and shape. For example, arguing that double an even number is even because an even number is a multiple of 2, and so that double this would also be a multiple of two. They might refer to Numicon as an example. When they argue that a statement is not true they need to find one example which contradicts it, called disproof by counter example. They might decide when some statements are always or sometimes true.
- Use the finding rules and describing patterns investigations to ask children to generate general statements and then explain why they are true.

Further support for guidance in reasoning can be found on:
https://www.ncetm.org.uk/resources/44672
Further activities can be found on:
www.nrich.maths.org
https://www.openmiddle.com/
Problems for the More Able (Y1-Y6, published 2006 but still relevant)

	Working systematically Finding all possibilities	**Generalising and conjecturing Explaining and justifying**	**Thinking strategically Interpreting information**	**Reasoning, convincing and proof Considering general**

	Enumerating possibilities for combinations	Finding rules and describing patterns	Solving logic problems	statements: "Convince yourself, convince your friend, and convince your enemy".
Year R	**Example Learning Outcomes:** •Talk about things being in order. •Identify same and different. •Use ordinal vocabulary, 1st 2nd etc. •Sort objects using and explaining criteria •Explain what they are thinking and doing. •Represent work with objects or pictures and discuss it, talk about ways to check that there are no omissions or repetitions	**Example learning outcomes:** •Talk about, recognise and recreate simple patterns. •Identify same and different. •Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices •Sort objects using criteria and explaining •Make a prediction about the next part of the pattern.	**Example learning outcomes:** •Recognise similarities and differences. •Sort objects using several criteria and sort to their own criteria, justifying their choices. •Say why an item does not belong into a set. •Guess the criteria being used to sort objects. •Explain what they are thinking and doing.	**Activities across the mathematics curriculum:** Explain why an answer is correct •Link to persuasive language
	Example activities: • Sorting activities • Billy the clown wears a coloured nose and bowtie for his show. He has a red nose and a blue nose. Make pictures of Billy with his noses. How many different pictures? He has a spotted bowtie and a striped bow tie. Make pictures of Billy with his bow ties. How many different pictures? Can you make different outfits for Billy? Use a nose and a bow tie. How many different outfits? • How many different ice creams can you make if you choose one scoop of either chocolate or strawberry ice cream? Now try with a plain or chocolate cone. • A lady bird has six spots. She can have some spots on the left and some on the right. Draw as many different ways of arranging the spots as you can. • Put ten things into 2 paper bags. Can you do it in a different way?	**Example activities:** • How many smarties in a pack? How many of each colour? Will it be the same for the next pack? Which colour is there more of? • Find different shaped sponges. Which one holds the most water? • Stand up 10 skittles. Have one go at knocking some down with a soft ball or bean bag. Record how many are still standing and how many you knock down. Can you guess how many were knocked down before you counted them? • Copying, making and talking about patterns with toys, bricks, beads etc • PNS Finding rules and describing patterns: Teddy's presents	**Example activities:** • How is your shoe different to your neighbour's? • PNS Logic problems: Shoes, nature sort • Solving everyday problems about classroom tasks e.g. do we have enough apples for snack time?	**Example activities:** When answering simple problems involving addition and subtraction in their play • why they have used particular shapes in junk modelling • why certain shapes fit into a jigsaw • explain how they work out doubles and halves using resources

	• PNS Finding all possibilities: In the café, working in sand, railway track			
	Working systematically Finding all possibilities Enumerating possibilities for combinations	**Generalising and conjecturing Explaining and justifying Finding rules and describing patterns**	**Thinking strategically Interpreting information Solving logic problems**	**Reasoning, convincing and proof Considering general statements: "Convince yourself, convince your friend, and convince your enemy".**
Year	**Example learning outcomes:** •Identify same and different.	**Example learning outcomes:**	**Example learning outcomes:**	**Activities across the mathematics curriculum:**

	•Record different answers in a systematic way, identifying why this is important and explaining how they have done this •Explain how answers differ. •Recognise that there is sometimes more than one possible answer to a problem. •Give examples that match a given statement and those that don't. •Talk about patterns in their lists / results.	•Describe and recreate simple patterns involving numbers, shapes or items. •Decide whether examples satisfy given conditions. •Describe ways of solving puzzles and problems, explaining choices and decisions. •Represent findings orally, using pictures or practically. •Make a prediction about the next part of the pattern and explain why. •Recognise a simple relationship •Make predictions and conjectures	•Use one piece of information and see what effect it has. •Check that the answer meets all of the criteria. •Solve a problem using given facts. •Sort objects, number or shapes and explain why an example does or does not fit into a group	Explain why an answer is correct for example: • showing how they know the multiples of two, five or ten using resources such as Numicon or a number line or square • why a number sentence is correct or incorrect using known facts or resources, • why adding or subtracting zero has no effect, • how they know what half or quarter of a quantity object or shape is
	Example activities: • How many different ice creams can you make if you choose one scoop of either chocolate or strawberry ice cream with a plain or chocolate cone? • Holly and Ivy are two of Santa's elves. Holly wears a red hat and a red tunic. Ivy wears a green hat and a green tunic. In the morning they get dressed in the dark. How many ways can Holly get dressed? • Make a tower of 6 cubes (or a snake or a train) using 2 colours. How many can you make? • Put ten things into 2 paper bags. How many different ways can you do it? •In Teddy Town, teddies are either red or yellow and they live in red or yellow houses. There are 4 teddies - 2 red and 2 yellow, and 4 houses - 2 red and 2 yellow. Can you match each teddy to a house so that the four pairs are all different from each other?	**Example activities:** • Whose pencil case holds the most? • Whose school bag holds the most? • How many ways can you make a ten using Cuisenaire rods? • PNS Finding rules and describing patterns: Teddy's presents • nrich http://nrich.maths.org/9009 http://nrich.maths.org/9014 http://nrich.maths.org/8972	**Example activities:** • Shape or number Sudoku 2x2, 3x3 grids • Give me an example of … and another… eg give me an example of an even number, and another…., a pair of numbers with a sum of ten , and another…etc • PNS Logic problems: Toys, Granny's garden • Nrich http://nrich.maths.org/9036	**Example activities:** • Convince a friend or enemy whether general statements are true or false, for example: •All triangles have 3 sides •When you add two numbers, you can change the order of the numbers and the answer will be the same •You can make 4 different two-digit numbers with the digits 2 and 3 •When you add 10 to a number the ones digit stays the same. •3 + 4 = 4 + 3 (Commutative law) •Odd one out: for example with 2D and 3D shape • Show me that … is the same as…. Eg show me that 3 + 4 = 4 + 3 • Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true. • http://nrich.maths.org/9016 (Link to persuasive language)

	• You buy a lollypop for 6p and give the exact money, how many different ways can you pay? • List numbers which total 10 • Billy the clown wears a coloured nose and bowtie for his show. He has a red nose and a blue nose, and a spotted bowtie and a striped bow tie. How many different outfits can he appear in? • PNS Finding all possibilities: Lollipops, down the path • Nrich http://nrich.maths.org/9798			
	Working systematically Finding all possibilities Enumerating possibilities for combinations	**Generalising and conjecturing Explaining and justifying Finding rules and describing patterns**	**Thinking strategically Interpreting information Solving logic problems**	**Reasoning, convincing and proof Considering general statements: "Convince yourself, convince your friend, and convince your enemy".**
Year	**Example learning outcomes:** •Use a systematic way to solve a	**Example learning outcomes**:	**Example learning outcomes:** •Solve a problem by identifying given	**Activities across the mathematics curriculum:** Explain why an answer is correct, for example:

	problem. •Create a systematic list of possibilities. •Talk about why it is a complete list and how they have been systematic. •Look for patterns and possible general statements or relationships	•Identify patterns and relationships involving numbers or shapes, and use these to solve problems. •Talk about how a pattern will continue and make predictions. •Talk about the pattern generally, discussing a general relationship or statement in words •Describe and explain methods, choices and solutions to puzzles and problems.	facts and prioritising them. •Identify necessary information for solving problems •Confirm that they have found the correct solution by checking in another way. •Use recording to help them make sense of the information given and to find missing information	• use known facts or inverse operations or place value or resources such as Dienes or Numicon or a number line to show why a number sentence is correct or incorrect, • use resources to show how they know how to find a fraction of a quantity or shape or object and that 2/4 = ½ • how they have compared and ordered items by measuring • why different combinations of coins might have the same value • why times expressed in different ways may be the same • how they solved problems using pictograms, tallies or block diagrams
	Example activities: • If three bears, a red bear, a yellow bear and a green bear, play each other at table tennis, each taking it in turns to play another bear, how many games will there be? • How many different football strips could you make choosing from 2 T-shirts and 2 pairs of shorts? • How many different numbers can you make with the digits 1, 2 and 3? • Arrange 3 different coloured Smarties in different ways • List pairs of number which have a ones digit of 3 when added together • List pairs of numbers with a difference of 3 • Use 7 cubes - 5 of them of one colour and 2 of another colour. These 7 have all to be joined together. The five that are of one colour must all touch the table that you are working on. The two that are of a different colour must NOT touch the table.	**Example activities:** • Make a family of multi-link animals, eg a baby dog: How many cubes? Make the next one in the dog family: How many cubes? Make the next members of the dog family How many cubes for each one? How many cubes for the 100th member? Can you see a patterns? How can you work out how many cubes for any dog in the family? • If you fill your pencil case with pennies how rich are you? What about 2pence pieces? 10 pence pieces? • How high is your chair? Your table? Your door? How high would they need to be for a giant child double your height? • If a bank only has 2p and 5p coins, what amounts can you make? • Make multi-link towers of the same size and put them on the	**Example activities:** • Give me an example of … and another… eg give me an example of a pair of numbers with a difference of 2, and another…., a multiple of 3, and another…etc • Shape or number Sudoku 3x3, 4x4 grids • PNS Logic problems: Shape puzzler, sandwich shop • Nrich http://nrich.maths.org/9036	**Example activities:** • Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true. • Convince a friend or enemy whether these statements are true or false. Explain their thinking, showing why a general statement may be true or not true with the use of particular examples. For example: When you subtract ten from a number, the ones digit stays the same You can add 9 to a number by adding 10 and subtracting 1 All even numbers end in 0, 2, 4, 6, 8 A cube has 9 faces If you have 3 digits, and use each one exactly once in a three-digit number, you can make 9 different three digit numbers • Odd one out activities eg looking at three numbers such as 2, 15, 30, decide which is the odd one out and convince your friend • Same and different activities eg 2D and 3D shapes • Show me that … is the same as…. Eg show me that 2 lotsof 5 is the same as 5 lots of 2

	How many different shapes can you find? • PNS Finding all possibilities: Maisie and the maze, line of symmetry • Nrich http://nrich.maths.org/9798	corners of a square. How many cubes did you use? Make your towers a different size but keep them all the same. How many now? Try with a triangle or a pentagon. • PNS Finding rules and describing patterns: Hop scotch grid • http://nrich.maths.org/9009 http://nrich.maths.org/9014 http://nrich.maths.org/8972		• Nrich http://nrich.maths.org/9016 (Link to persuasive language)
	Working systematically Finding all possibilities Enumerating possibilities for combinations	**Generalising and conjecturing Explaining and justifying Finding rules and describing patterns**	**Thinking strategically Interpreting information Solving logic problems**	**Reasoning, convincing and proof Considering general statements: "Convince yourself, convince your friend, and convince your enemy".**
Year 3	**Example learning outcomes:** •Prove that they have found all possible answers by being systematic. •Use patterns to make predictions about the number of combinations •Use patterns to talk about general statements or relationships	**Example learning outcomes:** •Generate patterns by considering examples systematically in an investigation •Make predictions based on patterns in results in an investigation •Make general statements and discuss	**Example learning outcomes:** •Solve a puzzle by identifying the facts and prioritising them. •Use one piece of information in the problem and see what effect it has. •Identify necessary information for solving problems	**Activities across the mathematics curriculum:** Explain why an answer is correct, for example: • use known facts or inverse operations or place value or resources such as dienes or a number line to show why a number sentence is correct or incorrect, • Use resources such as dienes and place value counters to show how they used column methods

		relationships using everyday language •Describe and explain methods, choices and solutions to puzzles and problems. •Continue more complex patterns.	•Check that their solution meets all the criteria.	for addition and subtraction, demonstrating that ten ones is one ten and ten tens is one hundred • Use resources to show how they know what one tenth of a number is • Use resources or pictures to show how they know what a fraction of a number is and to show equivalent fractions • How they know what the perimeter of a shape is • Why times expressed in different ways may be the same • How they use conversions between metric units of measurements to solve problems (eg m,, cm, mm, kg, g, l ml) • Why a full turn is the same as four quarter turns etc • How they solved problems using bar charts, pictograms and tables
	Example activities: • Billy the clown wears a coloured nose and bowtie for his show. He has a red nose and a blue nose, and a spotted bowtie and a striped bow tie. How many different outfits can he appear in? How many outfits if he buys a new nose and bow tie? • List trios of numbers which total 101 • List numbers which leave a remainder when divided by 5 • Find the shapes which straight sides which can be found by cutting a square in to two pieces • PNS Finding all possibilities: fireworks, Susie the snake • Nrich http://nrich.maths.org/9803	**Example activities:** • Draw a 2x2 square on a 100 square. Add the diagonals. What do you notice? Will it always be true? Try different shaped squares/ rectangles. • Make a net for a cube. How many different cube nets can you find? • Which numbers can you make using only four 3s and any combinations of operations? • PNS Finding rules and describing patterns: Hop scotch grid, Party bags, L-shaped models http://nrich.maths.org/8915 http://nrich.maths.org/8917 http://nrich.maths.org/8909	**Example activities:** • Give me an example of … and another… e.g., give me an example of a fraction equal to 1/2, and another...a pair of numbers which total 100, and another…etc. • Shape or number Sudoku, 3x3 grids and sets of 3x3 grids e.g., 9 x9 • PNS Logic problems: coloured shapes, Rebecca's school day • Nrich http://nrich.maths.org/8944	**Example activities:** • Convince a friend or enemy whether these statements are true or false. Explain their thinking, showing why a general statement may be true or not true with the use of particular examples. For example: Any odd number is one more than an even number Any even number can be made as the sum of two odd numbers The multiples of 4 are always even • Odd one out activities • Same and Different Activities • Show me that … is the same as…. E.g., show me that a litre is the same as two lots of 500 ml • Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true. • Nrich http://nrich.maths.org/8921 (Link to persuasive language)

	Working systematically Finding all possibilities Enumerating possibilities for combinations	**Generalising and conjecturing Explaining and justifying Finding rules and describing patterns**	**Thinking strategically Interpreting information Solving logic problems**	**Reasoning, convincing and proof Considering general statements: "Convince yourself, convince your friend, and convince your enemy".**
Year 4	**Example learning outcomes:** •Solve a problem by checking possible solutions against given criteria. •List possible answers in a systematic way efficiently. •Justify the approach as being systematic. •Prove that all items are listed •Make a general statement and provide a convincing argument that it is true. •Use a pattern to predict the next number of combinations	**Example learning outcomes:** •Report solutions to puzzles and problems, giving explanations and reasoning orally and in writing, using diagrams and symbols •Use patterns to make predictions and general statements. •Talk about the justification for the general statement. •Describe and continue more complex patterns. •Draw conclusions from investigations and explain their reasoning	**Example learning outcomes:** •Solve a problem by identifying and prioritising given facts and information, checking possible solutions against given criteria. •Identify necessary information for solving problems •Solve a problem by identifying and prioritising given facts and information.	**Activities across the mathematics curriculum:** Explain why an answer is correct, for example: • use known facts or inverse operations or place value or resources such as dienes or a number line to show why a number sentence is correct or incorrect • Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction, • Explain how they solved word problems: choosing operations and disregarding unnecessary information and checking their answers • Explain what they know about multiplying by 0 and 1, and dividing by 1

				• Use an array to explain how to find factors of a number, and how to multiply two- or three-digit number by a one-digit number using the distributive law • Use resources or diagrams to show equivalent fractions and how to find a non-unit fraction of a quantity or shape • how they use conversions between metric units of measurements to solve problems (eg km, m, hour, minute) • how they found the area of a shape • why analogue and digital, and 12 and 24 hour times might be the same
	Example activities: • How many different ice creams can you make if you choose one scoop of either chocolate or strawberry ice cream with a plain or chocolate cone? Add in other flavours of ice cream, different types of cone, and then chocolate or toffee sauce on top. • List numbers which leave a remainder of 1 when divided by 7 • PNS Finding all possibilities: sheep dog trials, 3 digits • Nrich http://nrich.maths.org/9803	**Example activities:** • How many squares on a chess board? • Add three consecutive numbers. What do you notice about the answer? Now try adding 5, 7, 9 consecutive numbers. • Find the number of vertices, faces and edges on some 3D shapes. Do you notice a pattern? Is there a separate pattern for prisms and pyramids? • PNS Finding rules and describing patterns: Party bags, L shaped models • Nrich http://nrich.maths.org/8915 http://nrich.maths.org/8917 http://nrich.maths.org/8909	**Example activities:** • Give me an example of … and another… eg give me an example of a rectangle with perimeter of 24cm, and another…., three consecutive numbers with an odd total, and another…etc • Think of a number……. Double it, add 15 subtract 3, halve it, take away the number you first thought of. Now I will read your mind, the answer is 6! Why does it work? Make up your own • PNS Logic problems: shape puzzle, boys and girls • Nrich http://nrich.maths.org/8944	**Example activities:** • Convince a friend or enemy whether general statements are true or false. Explain their thinking, showing why a general statement may be true or not true with the use of particular examples and mathematical patterns and properties. For example: Any odd number is double a number add 1 If you multiply a number by 10 the digits move one place to the left The number of lines of reflective symmetry in a regular polygon is equal to the number of sides of the polygon The sum of three odd numbers is odd • Odd one out activities • Same and different activities for example with 2D and 3D shapes • Show me that … is the same as…. E.g., show me that ¼ of 24 is 6 • Explain why odd numbers added to odd numbers have even totals, etc., • Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true. • Nrich http://nrich.maths.org/8921 (Link to persuasive language)

	Working systematically Finding all possibilities Enumerating possibilities for combinations	**Generalising and conjecturing Explaining and justifying Finding rules and describing patterns**	**Thinking strategically Interpreting information Solving logic problems**	**Reasoning, convincing and proof Considering general statements: "Convince yourself, convince your friend, and convince your enemy".**
Year 5	**Example learning outcomes:** •Find all possibilities by working systematically. •Prove all possibilities are listed •Recognise when reasoning is systematic and when it is not. •Identify a pattern to make a prediction of the number of possibilities. •Make a general statement and provide a convincing argument and apply this to other situations with similar or more combinations.	**Example learning outcomes:** •Generate patterns through systematic examples in an investigation identify and describe patterns using mathematical language •Accurately predict a later term in a pattern or sequence •Use a pattern to suggest and test general statements. •Provide a convincing argument for the general statement. •Draw conclusions from investigations and explain their reasoning using words, symbols or diagrams, as appropriate	**Example learning outcomes:** •Use one piece of information in more complex problems and see what effect it has. •Identify necessary information for solving problems •Check that the answer meets the criteria. •Choose and use a recording system to organise the given information independently. •Use appropriate language that is associated with this type of logic problem, e.g. 'If this … then this will change …'	**Activities across the mathematics curriculum:** Explain why an answer is correct, for example: • use known facts or inverse operations or place value or resources such as dienes or a number line to show why a number sentence is correct or incorrect • Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction, • Use an array to show the distributive law and use this to explain their written methods for long multiplication • Explain how they solved word problems: choosing operations and disregarding unnecessary information and checking their answers • Explain common factors and multiples using an array, number line or resources

				• Prove whether a number is prime or not using an array or resources or known facts • Use resources or diagrams to show equivalent fractions and how to add and subtract fractions with denominators which are the same or multiples of the same number • how they use conversions between metric units and between metric and imperial units of measurements to solve problems • how they use facts about angles at a point or making a straight line to solve problems • how they solve problems using line graphs and tables
	Example activities: • Billy the clown wears a coloured nose and bowtie for his show. He has a red nose and a blue nose, and a spotted bowtie and a striped bow tie. How many different outfits can he appear in? How many outfits if he buys a new nose and bow tie? What about if he decides to wear a hat as well and buys a yellow and an orange hat too? Shoes too? List the factors of for example 48, how do you know you have them all? • List the square numbers between 50 and 500 • Place each of the numbers 1 to 5 in a V shape so that the two arms of the V have the same total. How many different possibilities are there? What do you notice about all the solutions you find? Can you explain what you see? Can you convince someone that you have all the solutions? What happens if we use the numbers from 2 to 6? From 12 to 16? From 37 to 41? From 103 to 107? What can you discover about a V that	**Example activities:** • The Tower of Hanoi Move all the discs to the right hand tower. Only move one disc at a time. Never put a large disc on a smaller one. What is the smallest number of moves? Try different numbers of different-sized disks. • If you have 3 towns, and each one has one road to the others, how many roads? How many roads for 4, 5, 6, any number of towns? • Explore the digit roots of numbers. To find the digit root, add the digits together. If your total has more than one digit root, continue to add the digits together. When your total has one digit, this is the digit root. What do you notice? What are the digit roots of the multiples of 3? • PNS Finding rules and describing patterns: candle problem, sequence of models • Nrich http://nrich.maths.org/8915 http://nrich.maths.org/8917 http://nrich.maths.org/89109	**Example activities:** • Give me an example of … and another… e.g., give me an example of two fractions with a total of 2, and another…., a 3D shape with at least two triangular faces, and another…etc., • Andrea, Peter, Debra and Simon are each wearing one of black, red, yellow and green T-shirts. Use the following clues to find out which colour shirt each person is wearing: The red shirt is worn by one of the boys. Andrea and the girl who always wears black are in different schools. Simon's shirt colour has the same number of letters as his name • PNS Logic problems: nicknames, tea for two • Nrich http://nrich.maths.org/8944	**Example activities:** • Convince a friend or enemy whether these general are true or false or sometimes true. •Explain their thinking, with the use of particular examples and mathematical patterns and properties. For example: A multiple of 6 is a multiple of 2 and 3 The digits of multiples of nine add up to 9 The product of two consecutive numbers is even Angles on a straight line add up to 180 degrees • Odd one out activities e.g., 2D and 3D shape • Same and different activities e.g., 2D and 3D shape • Show me that … is the same as…. E.g., show me that 1/5 of 10 is the same as ½ of 4 • Show me why adding consecutive odd number from 1 makes square numbers e.g., 1 + 3 + 5 = 9 (picture proof) • Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true. • Nrich http://nrich.maths.org/8921 (Link to persuasive language)

	has arms of length 4 using the numbers 1–7? • PNS Finding all possibilities: ice creams, treasure hunt • Nrich http://nrich.maths.org/9803			
	Working systematically Finding all possibilities Enumerating possibilities for combinations	**Generalising and conjecturing Explaining and justifying Finding rules and describing patterns**	**Thinking strategically Interpreting information Solving logic problems**	**Reasoning, convincing and proof Considering general statements: "Convince yourself, convince your friend, and convince your enemy".**
Year 6	**Example learning outcomes:** •Identify a pattern to make a prediction of the number of possibilities. •Make a general statement with a convincing argument and apply this to other situations with similar or more combinations. •Express the general statement from an investigation using mathematical language, symbols and sometimes with algebra.	**Example learning outcomes:** •Construct and use a general statement in words then symbols (e.g. the cost of c pens at 15 pence each is 15c pence). •Draw conclusions from investigations and explain their reasoning •Express the general statement from an investigation using mathematical language, symbols and sometimes with algebra.	**Example learning outcomes:** •Identify necessary information for solving problems •Prioritise and use given facts to solve and check complex logic problems. •Ask 'What if…?' questions. •Recognise the effect of extensions such as 'What if …?' questions. •Create their own criteria for solving a logic problem in the context of a solved problem •Refine and extend problems to generate fuller solutions	**Activities across the mathematics curriculum:** Explain why an answer is correct, using concise argument, involving symbols, mathematical language, graphs or diagrams. For example: • use known facts or inverse operations or place value to show why a number sentence is correct or incorrect • Use resources such as Dienes and place value counters to show how they used column methods for addition and subtraction, • Use an array to show the distributive law and use this to explain long multiplication • Explain how they perform long and short division, using resources such as place value counters • Explain how they solved word problems: choosing operations and disregarding unnecessary information and checking their answers

				• Use resources or diagrams to show equivalent fractions and how to order, add, subtract and multiply fractions with different denominators and divide fractions by whole numbers • Explain how they solve ratio and proportion problems, perhaps using the bar method • Explain when they can use the formulae for area and volume of shapes • How to generate number sequences and the rule for sequences they have generated • How they express missing number problems algebraically • How they use conversions between metric units and between metric and imperial units of measurements (miles and km) to solve problems • How they use facts about angles in a shape, at a point or vertically opposite to solve problems • How they solve problems using pie charts and line graphs, and calculate and interpret mean
	Example activities: • How many ways can three children line up for assembly? Four children? Ten children? • List fractions with the same value as 0.01 • List sets of three numbers with a mean of 6 • List primes between 50 and 70 • If the final score at the end of a hockey match was 4-2, what could the score be at half time? • PNS Finding all possibilities: King Arnold, 4 by 4 • Nrich http://nrich.maths.org/9803	**Example activities:** • How many handshakes take place if 30 people in a room shake hands with each other exactly once? • Make a 3x3x3 cube out of 27 small cubes. Imagine dipping it into paint. How many small cubes have: 3 faces painted? 2 faces painted? 1 face painted? O faces painted? Investigate for 1x1x1, 2x2x2 and other sized cubes • Investigating regions: Draw a circle and put two dots anywhere on the circumference. Join these up with straight lines and count how many regions you make. Try other numbers of dots. • Which numbers have odd totals of factors? • PNS Finding rules and describing patterns: candle	**Example activities:** • Give me an example of … and another… e.g., give me an example of a fractions equivalent to 3/4, and another…., a fraction smaller than 1/10, and another…etc., • Crossing the bridge: Four friends need to cross a bridge. They start on the same side of the bridge. A maximum of two people can cross at any time. It is night and they have just one lamp. People that cross the bridge must carry the lamp to see the way. A pair must walk together at the rate of the slower person: Rachel: - takes 1 minute to cross Ben: - takes 2 minutes to cross George: - takes 7 minutes to cross Yvonne: - takes 10 minutes to cross The second fastest solution gets the friends across in 21 minutes. The	**Example activities:** • Convince a friend or an enemy that general statements are always, sometimes or never true. If never true, disprove by counter example. Use particular examples but recognise that arguments should be based on general mathematical patterns and properties. For example: If you add three consecutive numbers, the sum is three times the middle number Multiplying does not always make the answer larger Dividing a whole number by half makes the answer twice as big Rectangles always have two diagonals which meet at right angles • Odd one out activities e.g., 2D and 3D shape • Same and different activities: e.g., 2D and 3D shapes • Show me that … is the same as…. e.g., show me that 30% of 60 is the same as 60% of 30

		problem, sequence of models • Nrich http://nrich.maths.org/8915 http://nrich.maths.org/8917 http://nrich.maths.org/8909	fastest takes 17 minutes. Can you work out how it is done? • PNS Logic problems: Albert square, house points • Nrich http://nrich.maths.org/8944	• Explain why odd numbers multiplied by even numbers are odd etc., • Explain why opposite angles are equivalent • Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true. Nrich http://nrich.maths.org/8921 (Link to persuasive language)